Electric properties of materials

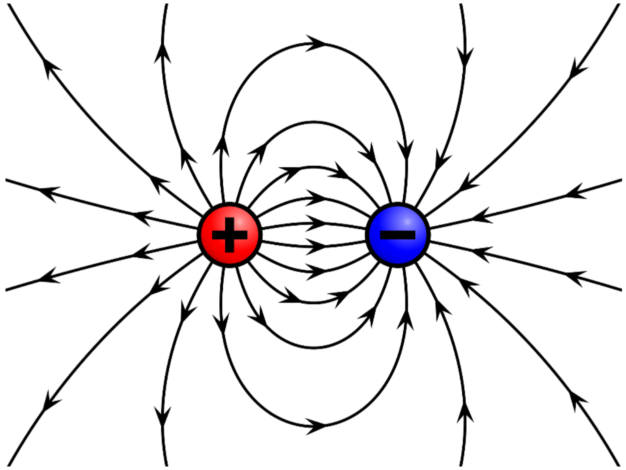
ver. 20191204

The electric field lines

Electric field: a modell which describes the effect of a charge in distance

Electric field strength (*E*) is defined at a given point as the limit of the electrical force per unit charge on an infinitesimal test charge.

$$E = \lim_{Q_t \to 0} F/Qt$$

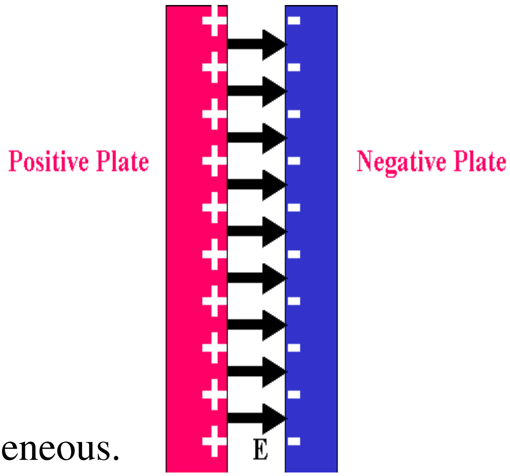


https://swissharmony.co.uk/wp-content/uploads/sites/8/2015/11/Electric-field.png

The **direction**: from the positive charge to the negative one.

Visualizations: by **electric field lines**.

Homogeneous field: has the same magnitude and direction at any place → the field lines straight and parallel



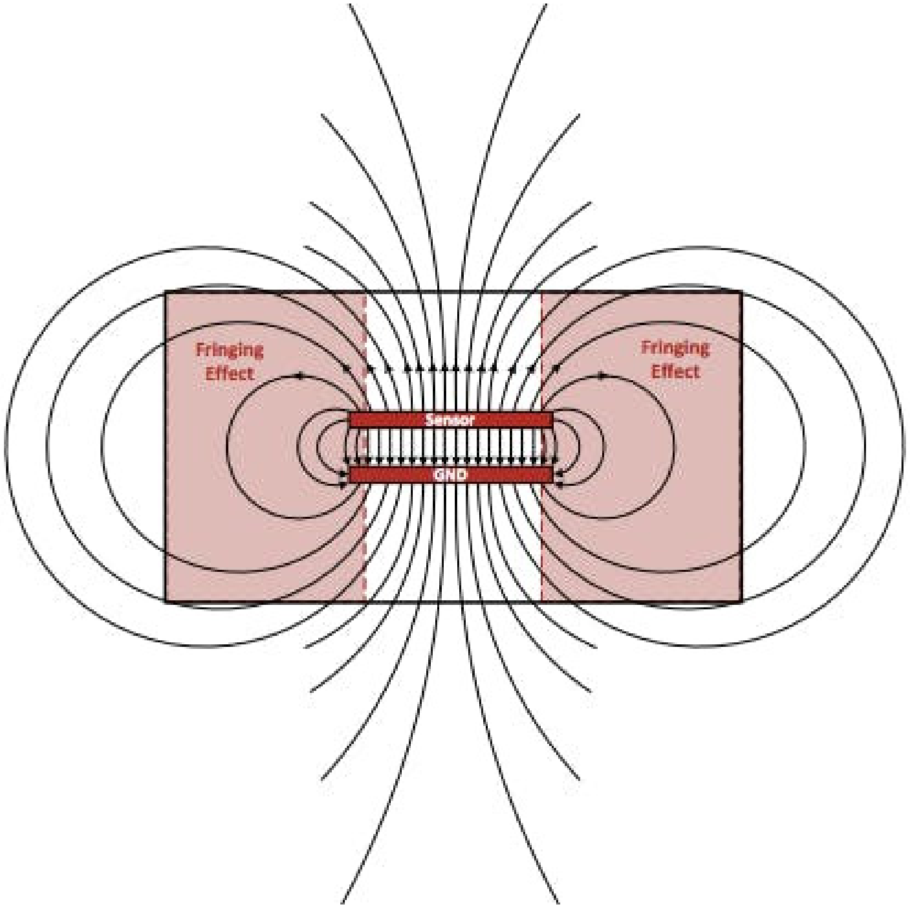
Electric field lines in an empty (vacuum) capacitor. The field inside is homogeneous.

The electric field lines

Experiments showing the electric field lines:

https://www.youtube.com/watch?v=63FnT0W-Hxc

The electric field lines are bent even at the edges of a condenser.



https://hackster.imgix.net/uploads/attachments/241068/eb9hkYA8yxJm6ok4Rsx9.png?auto=compress%2Cformat&w=680&h=510&fit=max

A parallel plate capacitor

Parallel plate capacitor: Consider two metallic plates of equal area *A* separated by a distance *d*.

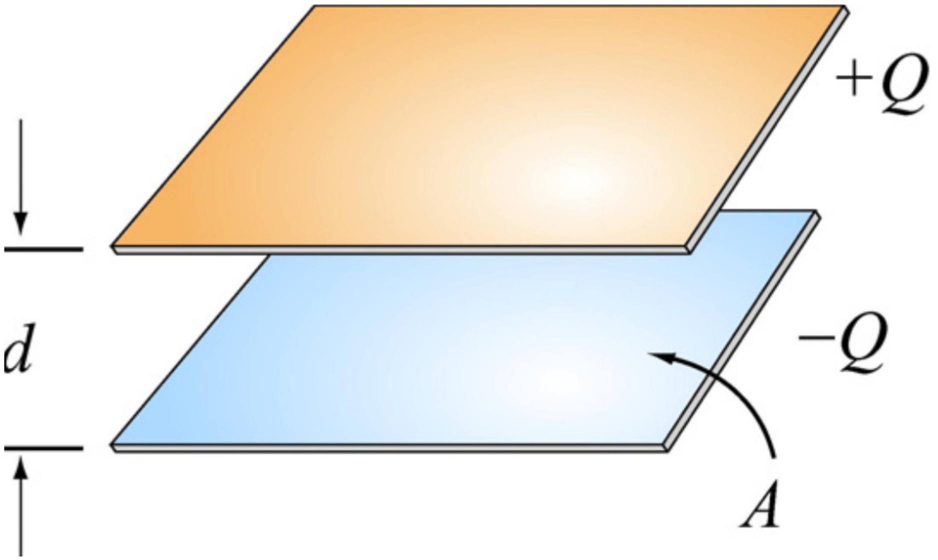
When $\Delta\varphi$ voltage is applied between the plates it charges them to +q and $-q$ charges, respectively. The proportionality constant between them is called **capacitance** (*C*): $q = C \cdot \Delta\varphi$

The SI unit of capacitance is F (farad), F = C/V.

The capacitance depends on what is inside the condenser.

The highest if there is nothing in (i.e. vacuum is).

If any insulator material (a dielectric) is between the plates the capacitance decreases.



Calculation of the capacitance

Problem: A parallel plate capacitor plates are charged to ±0.000124 C. We measure 4.512 V voltage. Calculate the capacitance of the condenser!

$q = C \cdot \Delta\varphi$, therefore $C = \frac{q}{\Delta\varphi} = \frac{0.000124\ \mathrm{C}}{4.512\ \mathrm{V}} = 0.00002748 \frac{\mathrm{C}}{\mathrm{V}} = 0.00002748\ \mathrm{F} = 27.48\ \mu\mathrm{F}$

The **capacitance** of a parallel plate capacitor **can be expressed by geometric parameters**:

$$C = \varepsilon \cdot \frac{A}{d}$$

where A is the surface of the plates, d is the distance of them, ε is the permittivity of the filling material.

Capatitors with dielectric

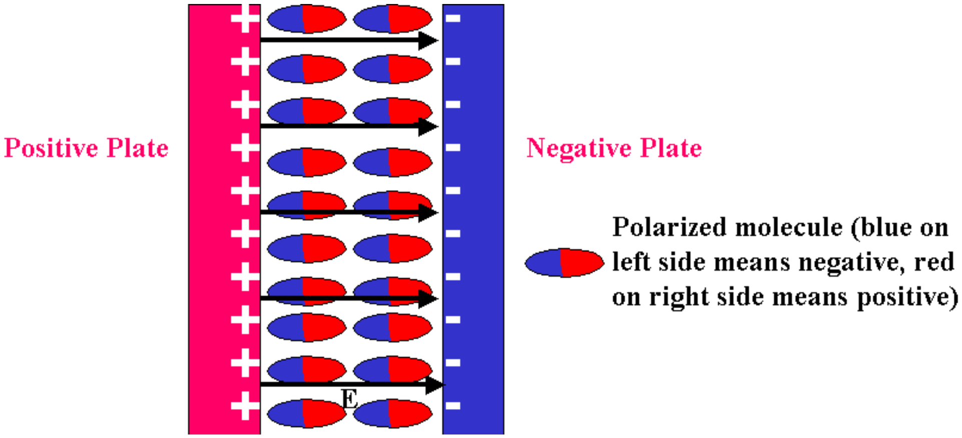
Dielectric: polarizable solid or liquid phase material which is good electric insulator. Dielectrics contain no mobile charged particle, e.g. alcohols, ethers, halogenated hydrocarbons.

Capacitors can be loaded with dielectric materials. When a dielectric is filled between the charged plates of a capacitor, the polarization of the medium produces an electric field opposing the field of the charges on the plate.

The electric field in the condenser with dielectric (called **effective electric field** (E_{eff}) is less than without dielectric:

$$E_{\text{eff}} = E - E_{\text{polar}}$$

where E is the electric field with vacuum, E_{polar} is the electric field caused by the dielectric.



The absolute and the relative permittivity

The **absolute permittivity** (ε_0) appears in the Coulomb's law and has significance in vacuum: $F = \frac{|Q_1 Q_2|}{4\pi\varepsilon_0 r^2}$

$$\varepsilon_0 = 8.85419 \cdot 10^{-12}\ \mathrm{F/m}$$

In any other material than vacuum the absolute permittivity should be replaced by the **permittivity** of the material (ε).

The ratio of the permittivity and the absolute permittivity is the **relative permittivity** (ε_r): $\varepsilon_\mathrm{r} = {}^{\varepsilon}/_{\varepsilon_0}$

The absolute and the relative permittivity

The relative permittivity **can be measured by the determination of capacitances**: $\varepsilon_\mathrm{r} = {}^{C}/_{C_0}$, where C and C_0 are the capacitances of the same condenser with the material and with vacuum in.

Relative permittivity of some materials (do not memorize the numbers!):

$\varepsilon_\mathrm{r,air} = 1.0006$, $\varepsilon_\mathrm{r,hexan} = 1.89$, $\varepsilon_\mathrm{r,cyclohexan} = 2.02$,
$\varepsilon_\mathrm{r,carbon\ tetrachloride} = 2.24$, $\varepsilon_\mathrm{r,diethylether} = 4.34$, $\varepsilon_{\mathrm{r,chloro}form} = 4.81$,
$\varepsilon_\mathrm{r,chlorobenzene} = 5.94$, $\varepsilon_\mathrm{r,liquid\ ammonia} = 15.5$, $\varepsilon_\mathrm{r,water} = 80$

Notes:

- $\varepsilon_\mathrm{r,air}$ is very close to ε_0
- $\varepsilon_\mathrm{r,water}$ is enormously large

Calculation of the permittivity

Problem: The capacitance of a parallel plate capacitor is 1.95 nF and it has the following geometric parameters: plate surface: 25 cm^2, plate distance: 0.0034 cm. Calculate the permittivity and the relative permittivity of the material filling the condenser!

Rearranging $C = \varepsilon \cdot \frac{A}{d}$ leads to $\varepsilon = C \cdot \frac{d}{A}$.

$$\varepsilon = C \cdot \frac{d}{A} = 1.95 \cdot 10^{-9}\ \mathrm{F} \cdot \frac{0.000034\ \mathrm{m}}{2.5 \cdot 10^{-3}\ \mathrm{m}^2} = 2.652 \cdot 10^{-11}\ \mathrm{F/m}$$

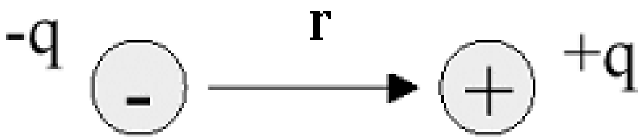
$$\varepsilon_\mathrm{r} = {}^{\varepsilon}\!/_{\varepsilon_0} = {}^{2.652 \cdot 10^{-11}\ \mathrm{F/m}}\!\Big/_{8.85419 \cdot 10^{-12}\ \mathrm{F/m}} = 3.00$$

The (electric) dipole moment

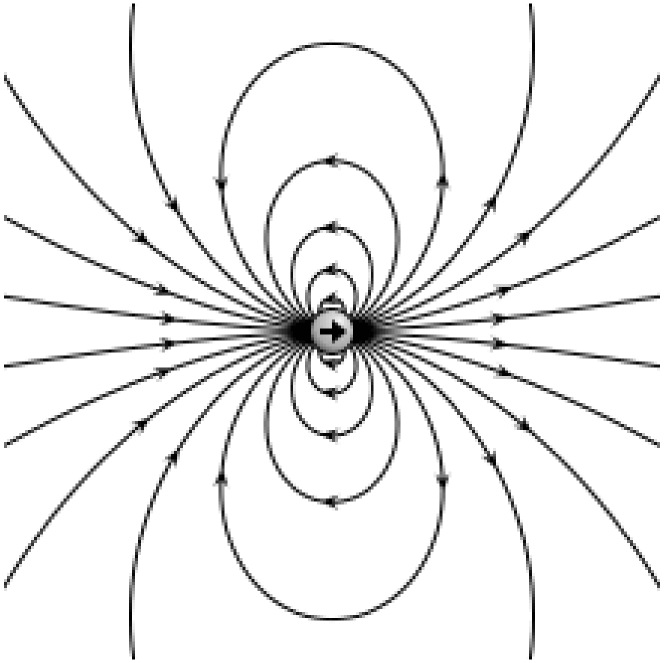
The electric dipole moment is a model for charge separation.
When two charges, +*q* and –*q* are separated by *r* distance the dipole moment ($\vec{p}$):

$$\vec{p} = q \cdot \vec{r}$$

The direction is from the negative charge to the positive charge.



SI unit: Debye (D). $1\ \mathrm{D} = 3.336 \cdot 10^{-30}\ \mathrm{Cm}$



Animation showing the electric field of an electric dipole. The dipole consists of two point electric charges of opposite polarity located close together. A transformation from a point-shaped dipole to a finite-size electric dipole is shown.
https://upload.wikimedia.org/wikipedia/commons/a/aa/VFPt_dipole_animation_electric.gif

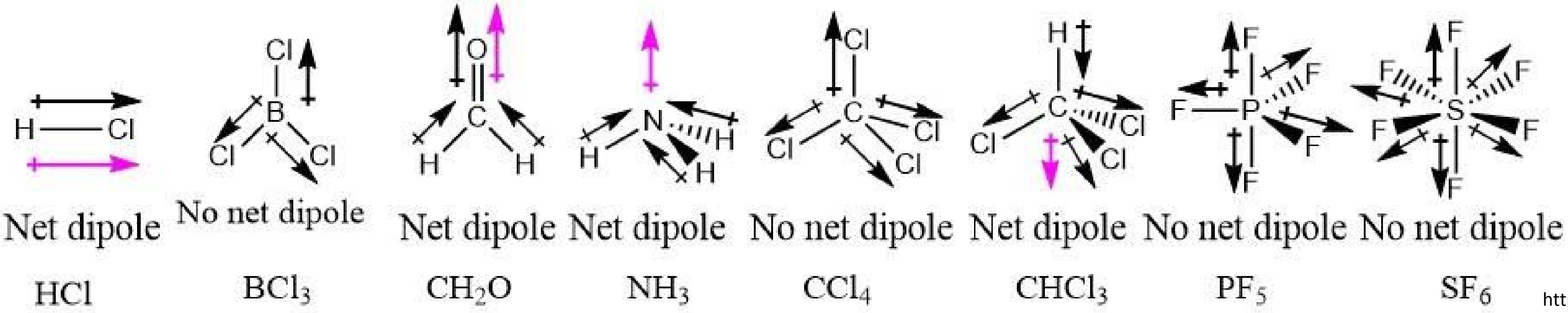
The permanent electric dipole moment

The electric dipole moment can be split to **permanent** and **induced** moments.

$$p = p_\mathrm{p} + p_\mathrm{i}$$

The permanent dipole moment of a two atomic molecule can be given using the definition of the dipole (i.e. given the charge separation and its distance).

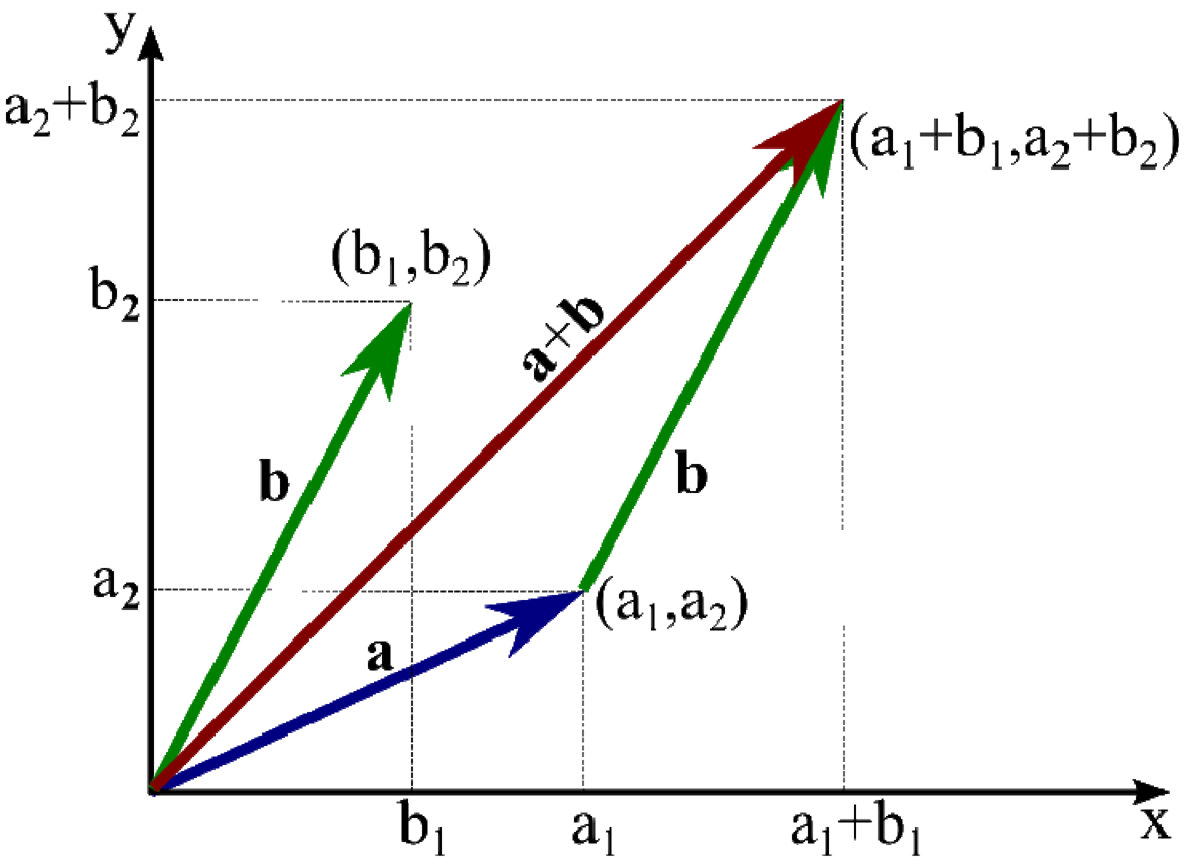
The permanent dipole moment of a **polyatomic molecule** can be calculated as the **vectorial sum of the bond dipole moments**.



All directions are the opposite!!!

https://i.stack.imgur.com/C8122.jpg

The vectorial sum



https://mathinsight.org/media/image/image/vector_2d_add.png

The induced electric dipole moment

Molecules having no permanent dipole moment are **nonpolar molecules.**

Nonpolar molecules may acquire dipole moment through interaction with the electric field. It is called **induced electric dipole moment**.

The induced dipole moment **exists only while the molecule is in electromagnetic field**.

Electric polarization

In elecromagnetic field the positive and negative charges (the atomic nuclei and the cloud of electrons) may change. This is called **electric polarization**.

Types of electric polarization:
- orientation polarization
- deformation polarization
  - electron polarization
  - atomic polarization (or ionic polarization)

Any type of polarization **leads to the formation of dipoles**.

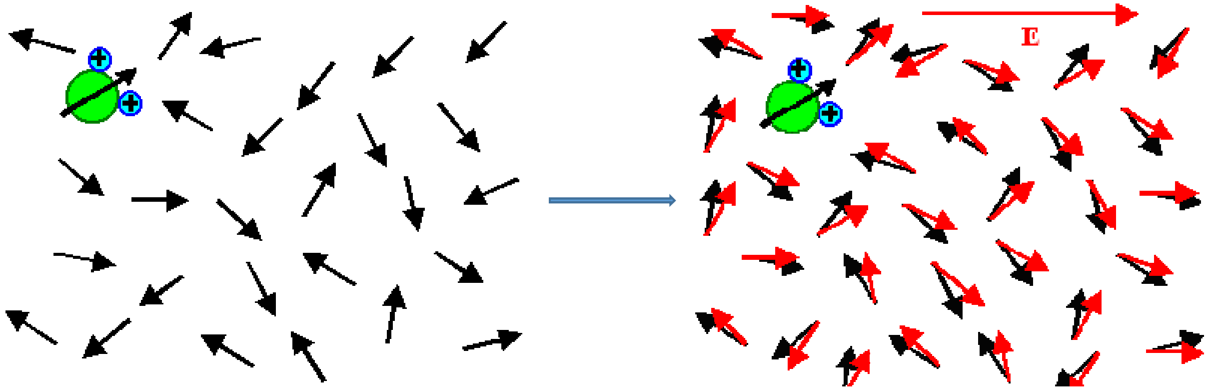
Quantitative measure of the polarization is the **polarization density** (or simlply polarization): $\vec{P} = \frac{\overrightarrow{p_\mathrm{r}}}{V}$, where $\overrightarrow{p_\mathrm{r}}$ is the resultant dipole (the vectorial sum of each species, $\overrightarrow{p_\mathrm{r}} = \sum_\mathrm{i} \overrightarrow{p_\mathrm{i}}$), V is the volume.

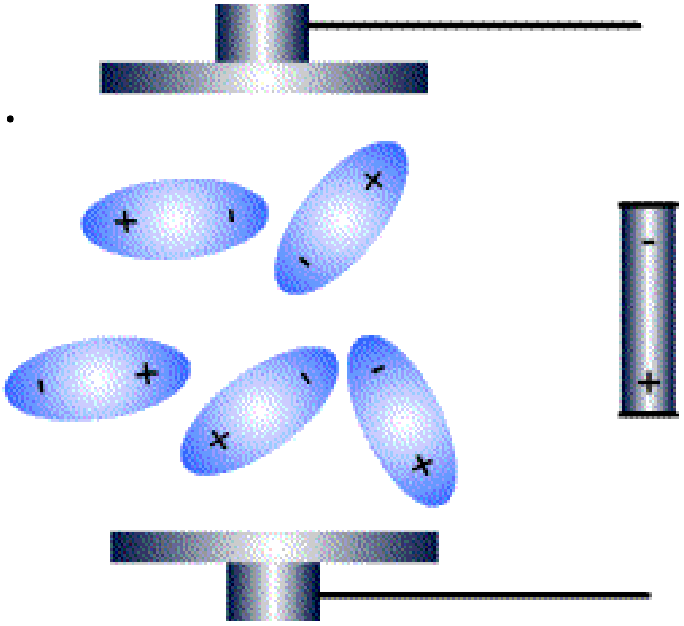
The orientation polarization

In elecromagnetic field the average orientation of dipoles becomes similar to the direction of the electric field (without *E* it is fully random).

This is called **orientation polarization** ($\overrightarrow{P_{\mathrm{Or}}}$).

The thermal agitation prevents the perfect orientation.



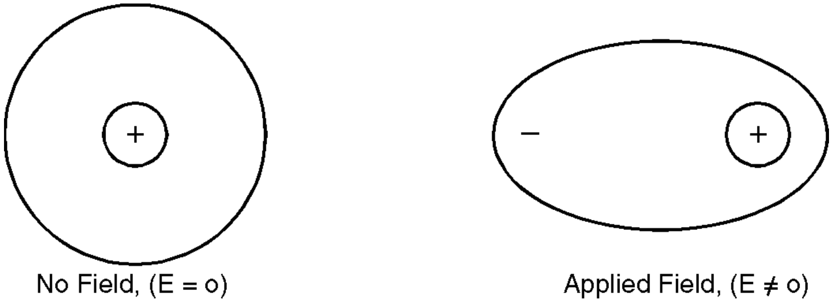


The deformation polarization

In elecromagnetic field the cloud of electrons or the position of nuclei may change. These are called **electron polarization** ($\overrightarrow{P_{\mathrm{E}}}$) and **atomic polarization** ($\overrightarrow{P_{\mathrm{A}}}$), respectively.

Depending on the field the cloud of electrons can be distorted without the change of the nuclei positions (the geometry of the molecule).

In ionic crystals the positions of ions may change (it is a kind of atomic polarization).



The total polarization

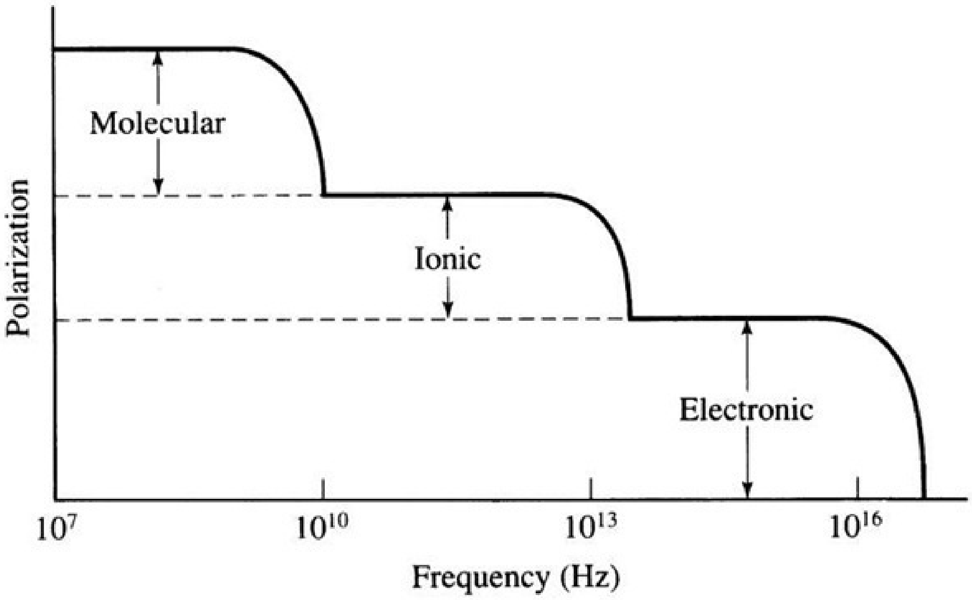
The total polarization is the sum of the different types of polarization:

$$\vec{P} = \overrightarrow{P_{\mathrm{E}}} + \overrightarrow{P_{\mathrm{A}}} + \overrightarrow{P_{\mathrm{Or}}}$$

In periodically changing electric field the different kind of polarizations appear at different frequencies.

The electromagnetic radiation causes periodically changing electric field. The dielectric material can absorb energy if **resonance conditions** are fulfilled.

Molecular movement (orientation) can follow only the slowest changes (low frequencies). Displacement of atoms can follow a bit larger frequency, but electrons are the fastest responsing ones.



MW

IR

VIS / UV

http://what-when-how.com/wp-content/uploads/2011/07/tmp838_thumb22.jpg

The absolute and relative refractive indeces

Refraction is the change in direction of a wave passing from one medium to another.

The quantitaive relation between the angles is described by **Snell's law**:

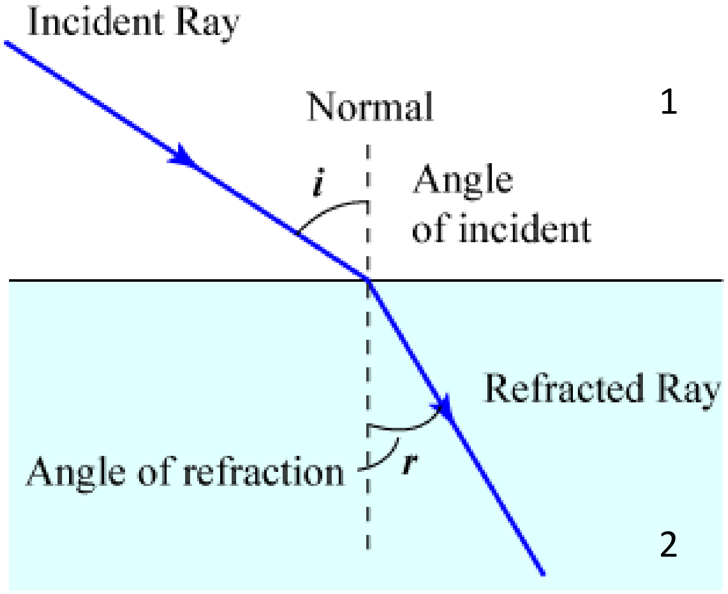
$$n_{2,1} = \frac{\sin i}{\sin r}$$

where $n_{2,1}$ is the **relative refractive index**.

If light comes from vacuum we can measure the **relative refractive index** (*n or* n_2):

$$n = \frac{\sin i_{\text{vacuum}}}{\sin r}$$

Note: do not mix refraction and reflexion!

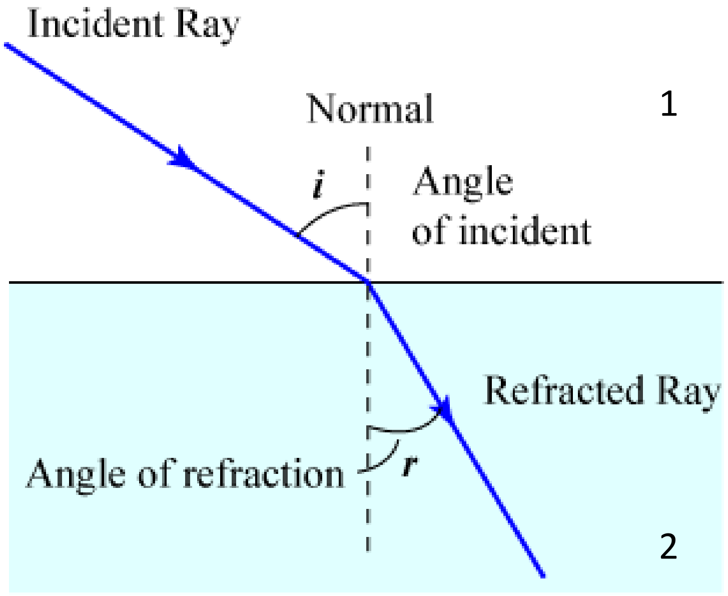


Expression of the refractive indeces by the light speed

The refractive **indeces** can be expressed by the speed of light in the material:

$$n_2 = \frac{c_0}{c_2} \quad \text{or} \quad n_{2,1} = \frac{c_1}{c_2}$$

where c_0, c_1 and c_2 are the speed of light in vacuum, medium 1 and 2, respectively.



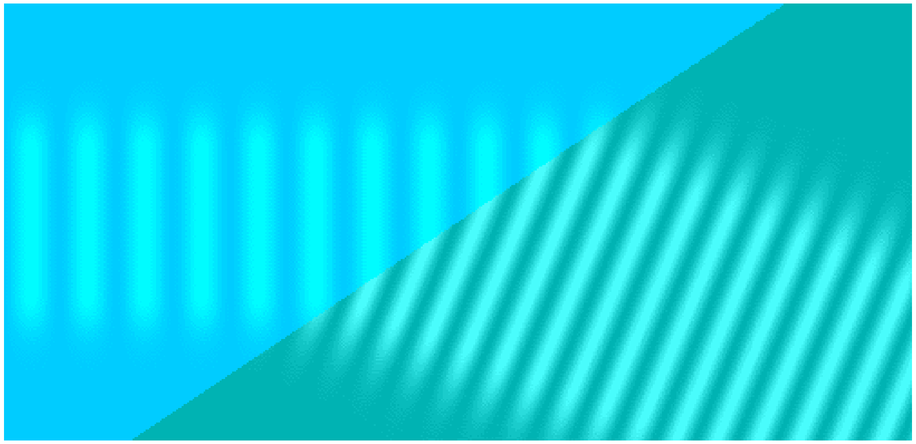
Combining these equations results: $n_{2,1} = \frac{n_2}{n_1}$

The medium having lower speed of light called **optically denser medium**.

The medium having higher speed of light called **optically rarer medium**.

General explanation of the refraction

When light changes medium one side of the wavefront is slower than the other. This asymmetrical speed of the light causes the change of angle. Once light is within the new medium with constant properties, it travels in a straight line again.



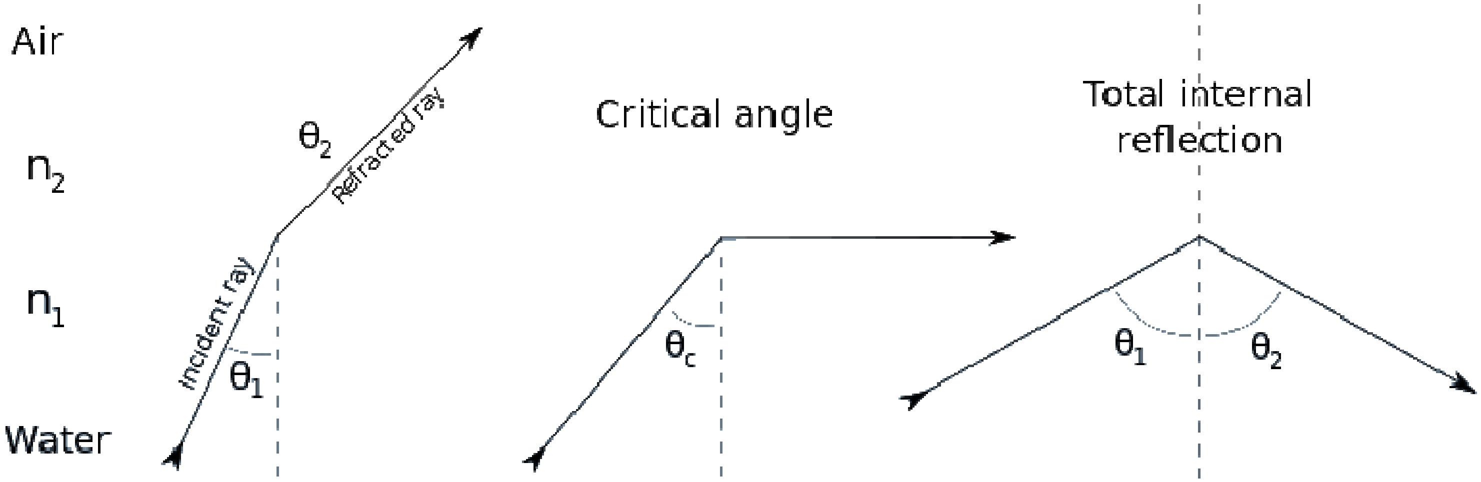
When a wave moves into a medium having lower speed of light the wavefronts get compressed. For the wavefronts to stay connected at the boundary the wave must change direction.

https://upload.wikimedia.org/wikipedia/commons/thumb/d/d4/Refraction_animation.gif/330px-Refraction_animation.gif

Total reflection and critical angle

If the light passing from a medium having higher optical density to a lower one ($n_{2,1} < 1$) the refractive angle can reach 90 degrees. In this case the incident angle is called the **critical angle** (θ_c). This case $n_{2,1} = \frac{\sin\theta_c}{\sin 90} = \sin\theta_c$

If the incident angle is **above the critical angle total internal reflection** occurs.

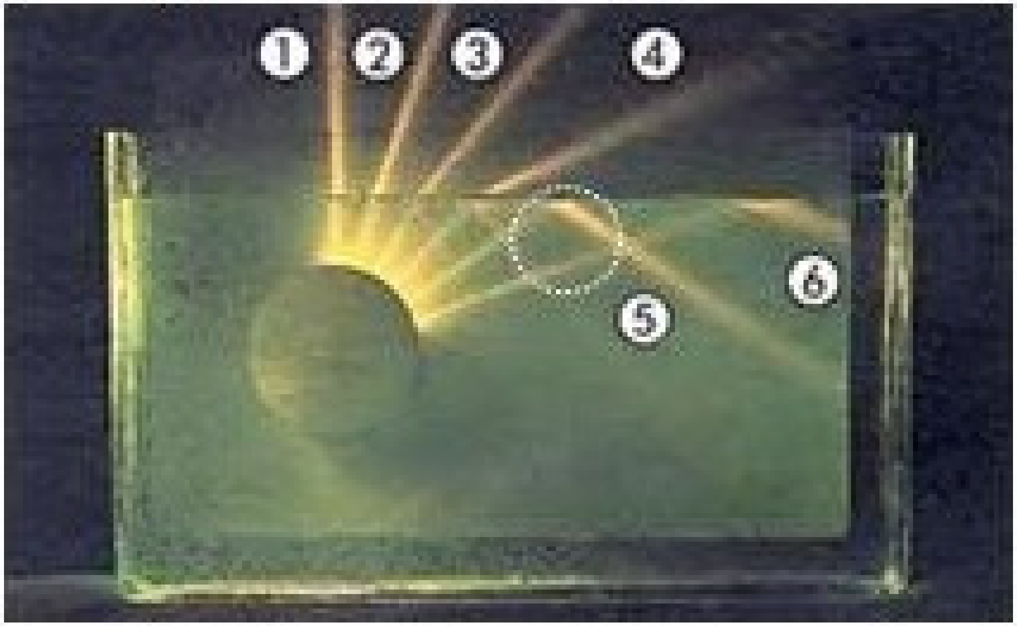


Visualization of different angle refractions and the total reflection

1) perpendicular light does’t refract

2), 3), 4) refracted rays

5), 6) total reflection



Simple calculations

Problem: The relative refractive index of two materials is 1.479. Calculate the refracted angle if the incident one was 17°, 70° and 90°! Give the critical angle!

$n_{2,1} = \frac{\sin i}{\sin r}$, therefore $\sin r = \frac{\sin i}{n_{2,1}}$

$$\sin r = \frac{\sin 17°}{1.479} = 0.19768, r = 11.4°$$

$$\sin r = \frac{\sin 70°}{1.479} = 0.63536, r = 39.4°$$

$$\sin r = \frac{\sin 90°}{1.479} = 0, r = 0°$$

There is no critical angle in this direction, since light goes to optical denser medium ($n_{2,1} > 1$).

Simple calculations

Problem: The absolute refractive indeces of two materials (*a* and *b*) are $n_a = 1.115$ and $n_b = 1.341$. Calculate the critical angle!

The relative refraction index must be less than 1 to have critical angle. It is fulfilled if light comes from *b* material to *a* material:

$$n_{a,b} = \frac{n_a}{n_b} = \frac{1.115}{1.341} = 0.83147$$

$\sin\theta_\mathrm{c} = n_{a,b}$, therefore $\theta_\mathrm{c} = \sin^{-1}\left(n_{a,b}\right) = 56.2°$

Variation of the refraction

The refractive index depends on:

- medium
- temperature (for accurate measurements we have to thermostat)
- wavelength / frequency

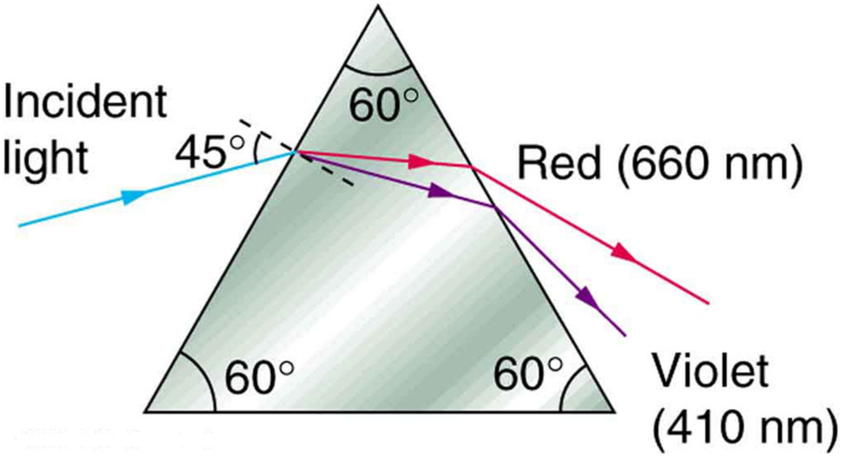
The **wavelength dependence** of the refractive index is called **dispersion**.

The dispersion

The **wavelength dependence** of the refractive index is called **dispersion**.

This can be described by empirical equations like the Cauchy's equation:

$$n_\lambda = A + B\frac{1}{\lambda^2} + C\frac{1}{\lambda^4}$$



https://cnx.org/resources/eb1fda307669dca7f61283704516cccd74d88f33/Figure%2026_05_06.jpg

Red light has lower energy, higher wavelength than violet. Therefore the second and third terms are smaller for red than violet, so $n_{\lambda,\mathrm{red}} < n_{\lambda,\mathrm{violet}}$

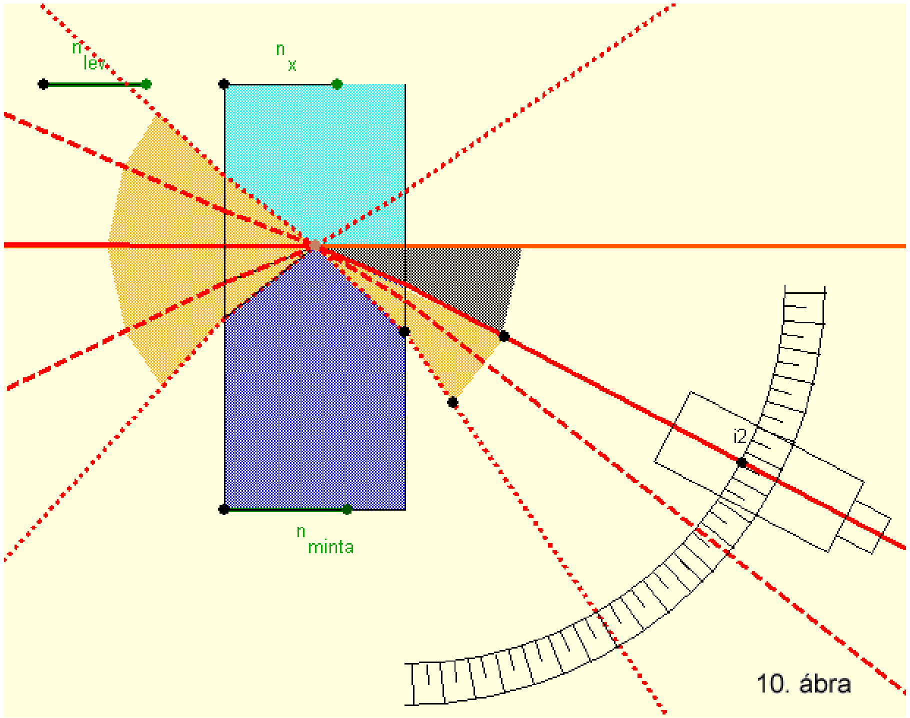
$$\sin r \cdot n_{2,1} = \sin i = \mathrm{constant}$$

Having the same incident angle (white light) red light has higher refraction angle.

Measurement methods of the refraction index

Methods can be based on the measurement of

- the critical angle (Abbe, Pulfrich)
- the angles of refraction (Pulfrich)
- interference (measures Δn)



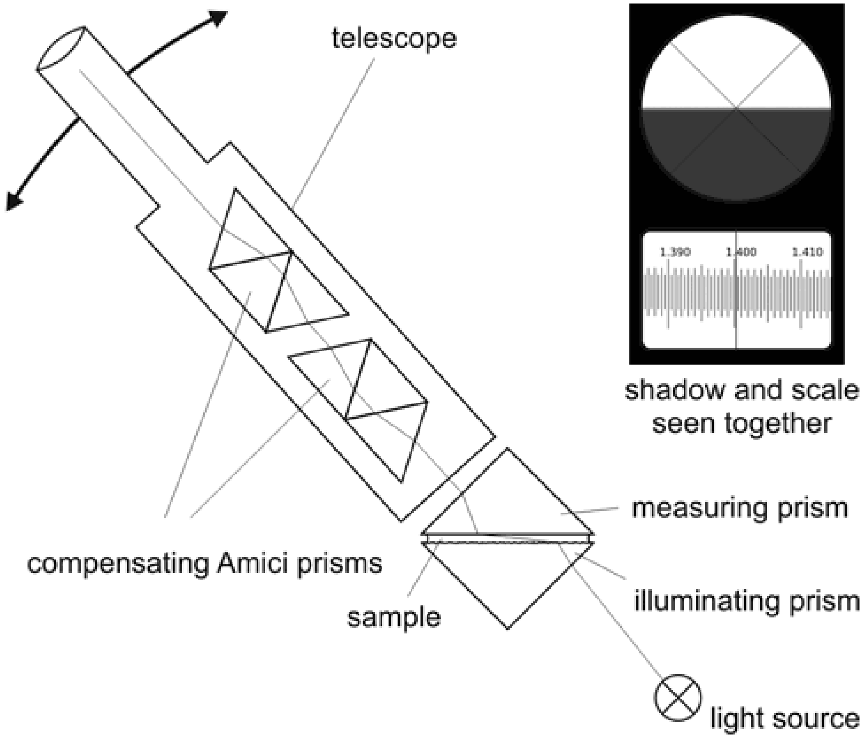
Angle measurement using Pulfrich refractometer

The Abbe refractometer

The sample is between two prisms (matted illuminating prism and measuring prism).

The telescope can be turned changing its angle.

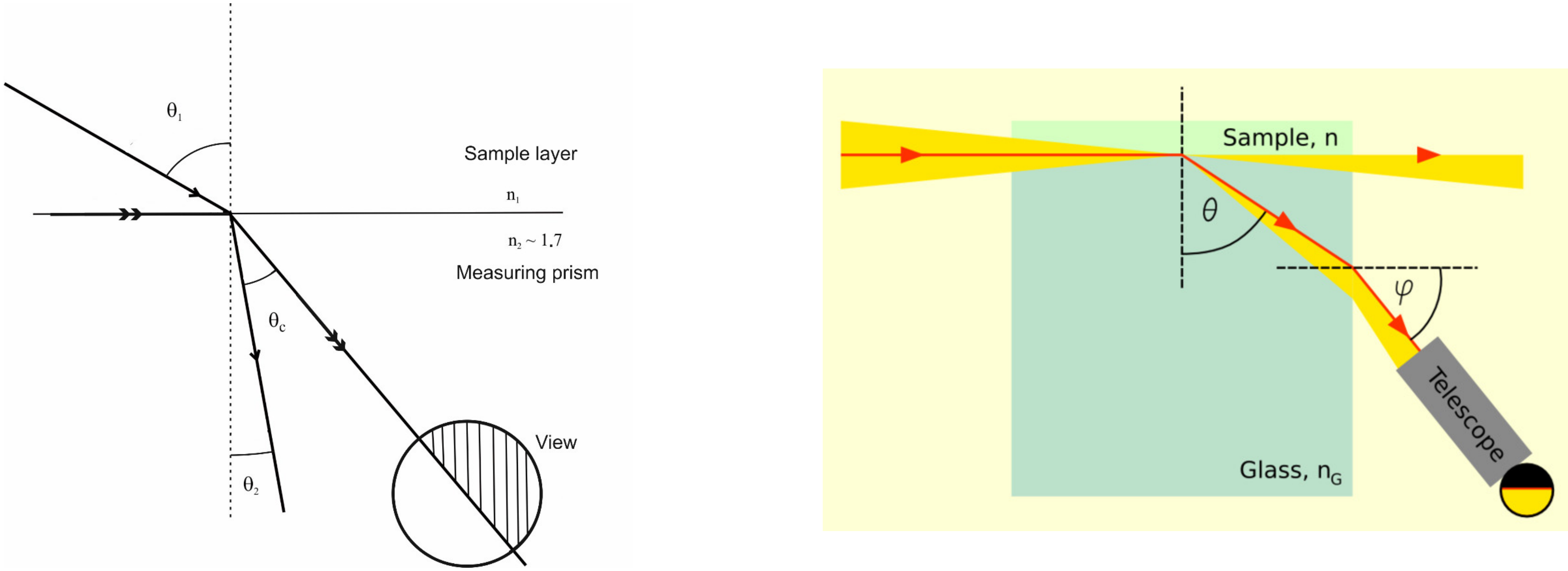
The dark and white border in the view must be positioned in the middle of the crosshair and the refractive index can be read from the scale.



http://www.refractometer.pl/img/abbe-refractometer-3.png

The working principle of the Abbe refractometer

Light coming in any angle refracts to smaller angles than the critical one.
Light coming in frictional fall angle reaches the critical angle, but there is no possibility of raytraces having larger refraction angles.
Therefore the **half of the view of the telescope will be light, the other half dark.**



Ray traces in the Abbe refractometer